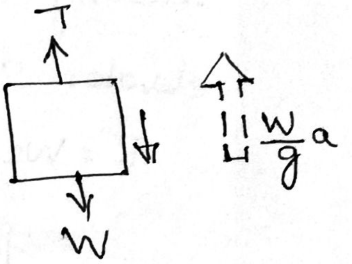
2. An elevator weighs 2500N and it moves vertically down with a constant acceleration. Write the equation for elevator cable tension. Starting from rest it travels a distance of 35m during an interval of 10 sec. Find the cable tension during this time. Neglect all other resistances to motion. What are the limitations of cable tension?



Net Force = W−T

$W-T = \frac{W}{g}a$ or $T - W + \frac{W}{g}a = 0$

$T = W\left(1-\frac{a}{g}\right)$

$T = W\left(1-\frac{a}{g}\right)$

$= 2500\left[1-\frac{a}{9.81}\right)$

It is the equation for cable tension.

$u=0$, $S=35m$, $t=10$ sec.

$S = ut + \frac{1}{2}at^2$

$a = \frac{2S}{t^2} = \frac{2\times 35}{100} = 0.70 m/s^2$

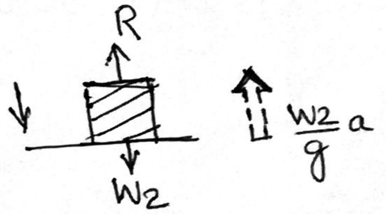
$T = 2500\left(1-\frac{0.70}{9.81}\right) = \underline{\underline{2321.61N}}$

When $a=0$, $T = 2500N$

$a_{max} = g$, $T = 0N$

Limits of cable tension $0 \to 2500N$

3. A cage carrying ten men of each 500N, starts moving downwards from rest in a mine shaft; cage attains a speed of 12m/s in 20m. Find force exerted by each man on the floor of the cage.



$W_2 = 500N$

$u = 0$

$V = 12m/s$.

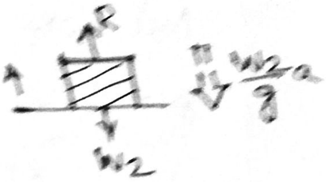
$S = 20m$

$V^2 = u^2 + 2aS$, $a = \frac{V^2}{2S} = \frac{12^2}{2\times 20} = 3.6 m/s^2$

$\downarrow \quad R - W_2 = W_2 \frac{a}{g} \qquad \therefore R = W_2\left(1 - \frac{a}{g}\right)$

$R = 500\left(1 - \frac{3.6}{9.81}\right) = \underline{\underline{316.51N}}$

4. An elevator weighing 5000N is ascending with an acceleration of 3m/s². During this motion its operator whose weight is 700 is standing on the scales placed on the floor. What is the scale reading. What will be the total tension in the cable of elevator during its motion.



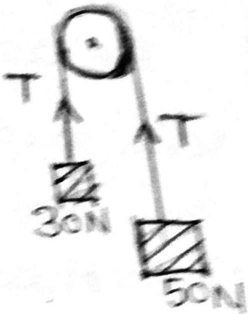
$R = W_2\left[1 + \frac{a}{g}\right]$

$= 700\left[1 + \frac{3}{9.81}\right] = 914.06N$

$T = W\left(1 + \frac{a}{g}\right) = 5700\left[1 + \frac{3}{9.81}\right] = \underline{\underline{7443.11N}}$

5. Two bodies of weight 50N and 30N are connected to the two ends of a light inextensible string which passes over a smooth weightless pulley, determine the acceleration of the system and tension in the string.

Consider the downward motion of 50N



Net Force $F = 50 - T$

By Newton's second law

$50 - T = \frac{50}{g}a \quad$ —①

Consider the upward motion of 30N

$F = T - 30$

$T - 30 = \frac{30}{g}a \quad$ —②

① + ② $\quad 50 - 30 = \frac{a}{g}(50 + 30)$

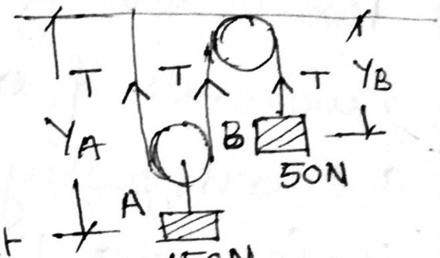
$a = \frac{9.81 \times 20}{80} = 2.45 m/s^2$

$T = 30\left[1 + \frac{2.45}{9.81}\right] = \underline{\underline{37.49N}}$

6. Determine the tension of the string and acceleration of blocks A and B weighing 150N and 50N connected by a string and a frictionless pulley as shown in figure.

Let L (Total length of the string)

$L = 2y_A + y_B$ — ①



Differentiating ① w.r.to time

$0 = 2V_A + V_B$

Again differentiating

$= 2a_A + a_B$

$\therefore a_A = -\frac{a_B}{2}$ (− opposite direction of motion)

If $a_B = a$, $a_A = -a/2$

Block A being heavier moves downwards, block B will move upwards.

Considering the upward motion of B

Net Force $= T - 50$

By Newton's second law $F = ma$

$T - 50 = \frac{50}{g} a$ — ①

Consider block A

Net Force $= 150 - 2T$ $\qquad a_A = \frac{a}{2}$

$150 - 2T = \frac{W}{g}\left(\frac{a}{2}\right)$

$= \frac{150}{g}\left(\frac{a}{2}\right)$

$150 - 2T = \frac{75a}{g}$ — ②

$2 \times ① \Rightarrow 2T - 100 = \frac{100}{g} a$ — ③

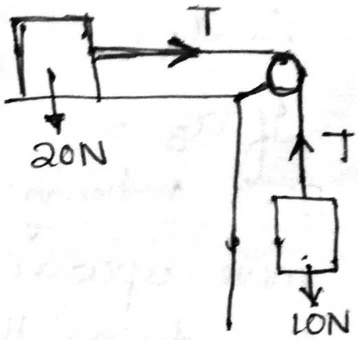
③ + ② $\Rightarrow 150 - 100 = \frac{a}{g}(100 + 75)$

$a = \frac{50}{175} \times 9.81 = 2.8\ m/s^2 = a_B$

$\therefore a_A = \frac{2.80}{2} = 1.4\ m/s^2$

$① \rightarrow T = \frac{50 \times 2.8}{9.81} + 50 = \underline{\underline{64.28N}}$

7. Two bodies of weight 20N and 10N are connected to the two ends of a light inextensible string passing over a smooth pulley. Weight 20N is placed on a horizontal surface and 10N is hanging free in air. Find acceleration of system and tension of the string
i) when the surface is smooth
ii) when the surface is rough $[g = 9.8\ m/s^2,\ \mu = 0.3]$



When the surface is smooth

Consider 10N moving downwards

$10 - T = F$ By Newton's II law

$10 - T = \frac{10}{g} a$ — ①

Consider the motion of 20N rightwards

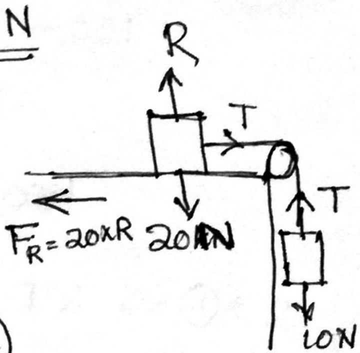
$F = T$

By Newton's 2nd law, $T = \frac{20}{g} a$ — ②

$① + ② \Rightarrow 10 = \frac{a}{g}(10 + 20)$

$a = \frac{10 \times 9.8}{30} = 3.26\ m/s^2$

$② \Rightarrow T = \frac{20}{9.81} \times 3.26 = \underline{\underline{6.65N}}$



When the surface is rough

Consider 10N moving down

$F = 10 - T$

$10 - T = \frac{10}{g} \times a$ — ① (Newton's 2nd law)

For 20N, Net $F = T - (20 \times 0.3)$

$T - 20 \times 0.3 = \frac{20}{g} a$ — ②

$$① + ② \Rightarrow 10 - 20 \times 0.3 = \frac{a}{g}(10+20)$$

$$a = \frac{10-6}{30} \times 9.8 = 1.31\,m/s^2$$

The tension of the string

$$① \rightarrow 10 - T = \frac{10}{g} \times 1.31$$

$$T = 10 - \frac{10 \times 1.31}{9.8} = \underline{\underline{8.66N}}$$

8 Two bodies of weight 40N and 50N are connected to the two ends of an inextensible string passing over a smooth pulley. The weight 40N is placed on a smooth inclined plane and 50N is hanging free in air. If the inclination of the plane is 15°, determine the acceleration of the system and tension in the string.

If in the above problem, with inclined surface rough having μ b/w the weight 40N and the inclined surface is 0.2, determine the acceleration of the system and tension in the string. Also calculate the distance moved by the weight 50N in 3 sec. starting from rest.

Case I. Surface smooth

Consider 50N ↓

Net Force, $F = 50 - T$

$$50 - T = \frac{50}{g} a \quad -①$$

Consider the upward motion of 40N.

$$F = T - 40\sin 15$$

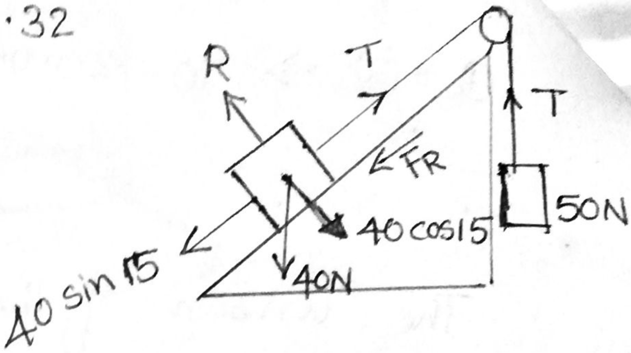
$$T - 40\sin 15 = \frac{40}{g} a \quad -②$$

$$① + ② \Rightarrow 50 - 40\sin 15 = \frac{a}{g}(50+40)$$

$$a = \frac{9.81\,[50 - 40\sin 15]}{90} = 4.32\,m/s^2$$

From eqn ① $T = 50 - \frac{50}{9.81} \times 4.32$

$= \underline{27.98N}$



Case II - Surface rough.

Consider 50N ↓

$F = 50 - T$

$50 - T = \frac{50}{g}a$ — ①

Consider the 40N ↑

$F = T - 40\sin 15 - \mu R$

$= T - 40\sin 15 - 0.2 \times 40\cos 15 = \frac{40}{g}a$ — ②

① + ② → $50 - 40[\sin 15 + 0.2\cos 15] = \frac{a}{g}(50 + 40)$

$a = 9.81\left[\frac{50 - 40(\sin 15 + 0.2\cos 15)}{90}\right] = 3.48\,m/s^2$

$T = 50 - \frac{50}{9.81} \times 3.48 = \underline{32.26N}$

Distance travelled by 50N in 3sec.

Given $u = 0$, $a = 3.48\,m/s^2$, $t = 3$ sec.

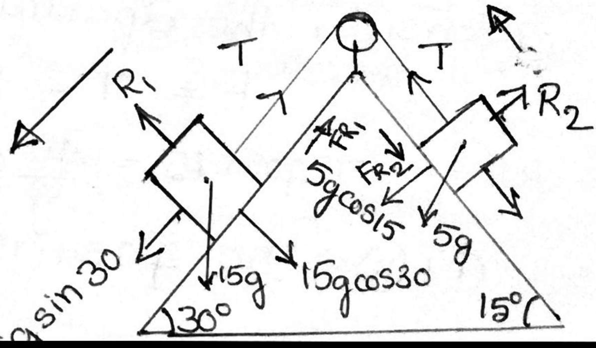
$S = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 3.48 \times 9 = \underline{15.66m}$

9. Two rough planes inclined at 30° and 15° to the hzl. are placed back to back. The masses 15kg and 5kg are placed on the inclined surfaces and connected by a string as shown in figure. If the coefficient of friction is 0.3, Find the resulting acceleration

Consider the downward motion of 15kg

$F = 15g\sin 30 - T - \mu \times 15g\cos 30$

$= \frac{15}{g}a$ — ①



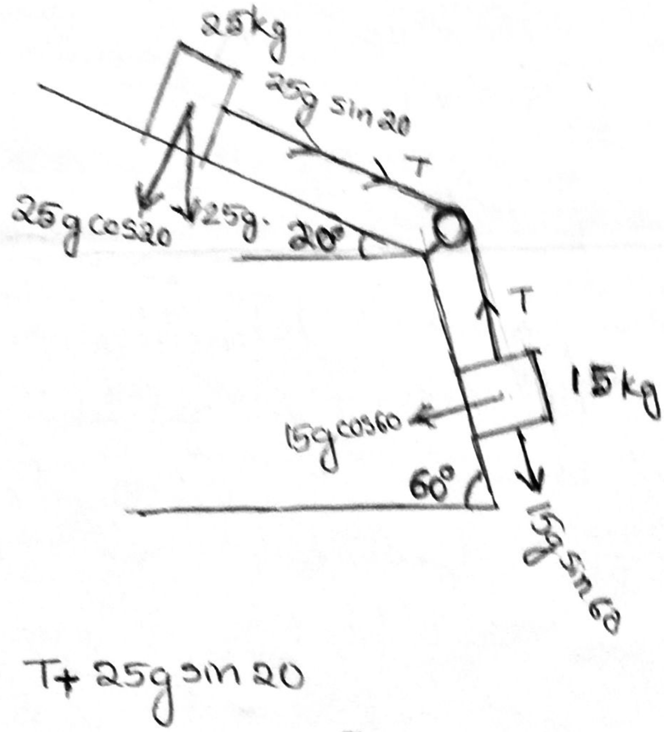
Consider the upward motion of 5g

$F = T - 5g\sin 15 - \mu \times 5g\cos 15 = 5a$ —②

①+② → $g(15\sin 30 - 4.5\cos 30 - 5\sin 15 - 1.5\cos 15] = a(15+5)$

$a = \frac{g}{20}(15\sin 30 - 4.5\cos 30 - 5\sin 15 - 1.5\cos 15] = 0.421 m/s^2$

10. Two bodies A and B are connected by a light inextensible string as shown in figure. If both the bodies are released simultaneously, what distance do they move in 3 sec. Neglect the friction between the inclined surfaces and two bodies.



25g ↓

Net Force, $F = T + 25g\sin 20$

$T + 25g\sin 20 = 25a$ —①

15g ↑, $F = 15g\sin 60 - T$

$15g\sin 60 - T = 15a$ - ②

①+② ⇒ $15g\sin 60 + 25g\sin 20 = (25+15)a$

$a = 5.28 m/s^2$

∴ Distance $= ut + \frac{1}{2}at^2$ $u=0$, $t = 3sec.$

$= \frac{1}{2} \times 5.28 \times 3^2$

$= \underline{23.76m}$

1

1. A particle is projected at an angle of 60° with the horizontal. The horizontal range of the particle is 5 kilometers. Find (i) the velocity of projection, and (ii) the maximum height attained by the projectile.

$\alpha = 60°$, $R = 5km = 5000m.$

Let u = velocity of progection

$R = \frac{u^2}{g} \times \sin 2\alpha$ ie $5000 = \frac{u^2}{9.81} \times \sin(2\times60)$

$u^2 = \frac{5000 \times 9.81}{\sin 120°} = 53639.7$ $\therefore u = 237.99 m/s$

$h_{max} = \frac{u^2 \sin^2\alpha}{2g} = \frac{237.99^2 \times \sin^2 60}{2\times9.81} = 2164.98m$

2. $T = \frac{2u\sin\alpha}{g} = \frac{2\times 237.99 \times \sin 60}{9.81} =$

2. A particle is projected at such an angle with the horizontal that the horizontal range is four times the greatest height attained by the particle. Find the angle of projection.

ie $R = 4 \times h_{max}$ -①

$R = \frac{u^2 \sin 2\alpha}{g}$, $h_{max} = \frac{u^2 \sin^2\alpha}{2g}$

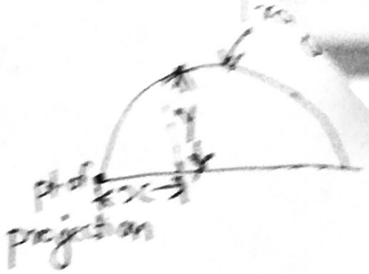
① → $\frac{u^2 \sin 2\alpha}{g} = 4 \times \frac{u^2 \sin^2\alpha}{2g}$

$\sin 2\alpha = 2\sin^2\alpha$

$2\sin\alpha\cos\alpha = 2\sin^2\alpha$ $\therefore \cos\alpha = \sin\alpha$

$\cancel{\cos\alpha}\ \frac{\sin\alpha}{\cos\alpha} = 1$ or $\tan\alpha = 1.0$ $\therefore \alpha = \underline{45°}$

27/9/ CSB. 3. A particle is projected with a velocity of 20m/s in air at an angle 'α' with the horizontal. The x and y co-ordinates of a point lying on the trajectory of the particle w.r.to point of projection are 20m and 8m resp. Find the angle of projection of the particle.



$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$8 = 20\tan\alpha - \frac{9.81\times 20^2}{2\times 20^2\cos^2\alpha}$$

$$= 20\tan\alpha - 4.905\sec^2\alpha$$

$$8 = 20\tan\alpha - 4.905(1+\tan^2\alpha)$$

$$4.905\tan^2\alpha - 20\tan\alpha + 12.905 = 0$$

$$\tan\alpha = 3.273 \ \& \ 0.8037$$

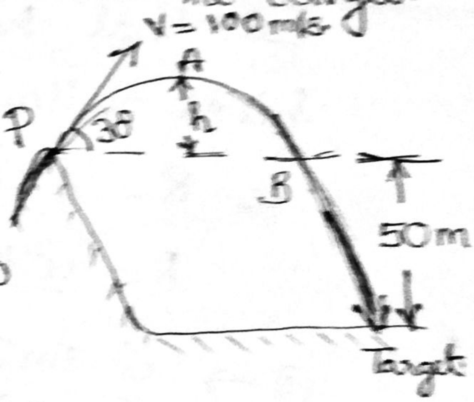
$$\therefore \alpha = 73^\circ\ 8' \quad \text{or } \alpha = 38^\circ 47'$$

4. A soldier positioned on a hill fires a bullet at an angle of 30° upwards from the horizontal. The target lies 50 m below him and the bullet is fired with a velocity of 100m/s. Determine

a) the maximum height to which the bullet will rise above the position of the soldier

b) the velocity with which the bullet will hit the target.

c) the time required to hit the target.



Initial velocity in the hzl. direction

$$U_x = 100\cos 30 = 86.6\text{ m/s}$$

Initial velocity in the vtl. direction $U_y = 100\sin 30$

$$= 50\text{ m/s}$$

a) $$h_{max} = \frac{u^2\sin^2\alpha}{2g} = \frac{100^2\times\sin^2 30}{2\times 9.81} = 127.42\text{ m}$$

b) The hzl. component of velocity remains constant

The vtl. component of of velocity when it hits the target V'_y

$$v^2 - u^2 = 2as$$

$u = V_y = 50$ m/s. $\quad a = -g \quad S = -50$m.

$$V'^2_y - V_y^2 = 2\times -9.81\times 50 \qquad \text{ie } V'^2_y - 50^2 = -2\times 9.81\times 50 = \pm 981$$

$$V'_y = \sqrt{50^2 + 9.81\times 2\times 50} = \pm 59\text{ m/s}$$

The velocity $V' = \sqrt{(V'_x)^2 + (V'_y)^2} = \sqrt{86.6^2 + 59^2} = \underline{104.8\text{ m/s}}$

c) $V = u + at$

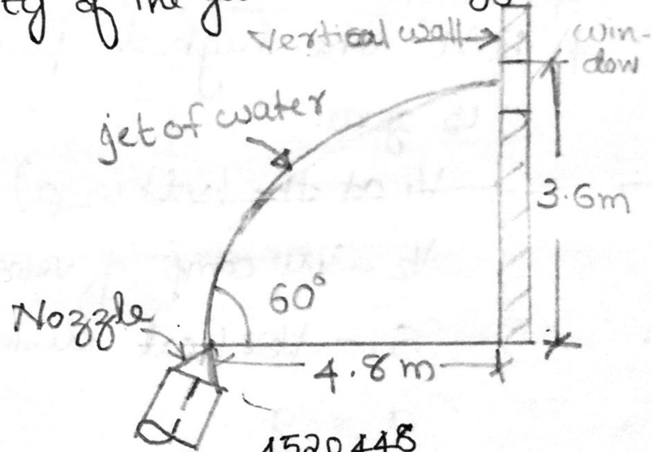
ie $V_y = 50\,m/s$ changes to $V'_y = -59\,m/s$ during downward motion

$$t = \frac{V-u}{a} = \frac{50 - -59}{9.81} = \underline{11.1\,s}. \quad t = \frac{-59-50}{9.81} = \underline{11.1\,sec}$$

The hzl. range $= u\cos\alpha \times t$

$$= 100\cos 30 \times 11.1 = \underline{961.28\,m}$$

5. A fireman holding a nozzle at a horizontal distance of 4.8m from a vertical wall, wishes to send a jet of water through a small window in the wall located at 3.6m vertically above the nozzle. If the inclination of the jet with the horizontal is 60° at the nozzle, calculate the required velocity of the jet at the nozzle exit



Horizontal distance, $x = 4.8m$

Vertical distance, $y = 3.6m$

Angle of projection, $\alpha = 60°$

Let u reqd. velocity of projection,

$$y = x\tan\alpha - \frac{g \times x^2}{2u^2 \times \cos^2\alpha}$$

$$3.6 = 4.8\tan 60° - \frac{9.81 \times 4.8^2}{2u^2 \times \cos^2 60} = 8.3136 - \frac{4520448}{u^2}$$

$$\frac{4520448}{u^2} = 8.3136 - 3.6 = 4.7136$$

$$u^2 = \frac{4520448}{4.7138} = 95.9$$

$$u = \underline{9.78\,m/s}$$

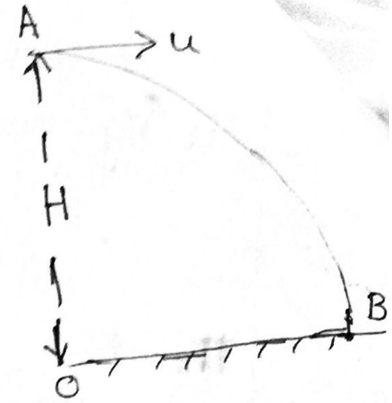
6. An air-craft moving horizontally at a speed of 720 kmph at a height of 1000m towards a target on the ground, released a bomb which hits the target. Find:

i) time required for the bomb to reach the target on the ground

ii) the horizontal distance of the air-craft from the target when it released the bomb.

Speed = 720 kmph = 200 m/s.

∴ Horizontal velocity of bomb $u = 200\ m/s$.



Height $H = 1000\ m$.

i) Let t time to hit the target

$$H = \frac{1}{2} g t^2$$

$$1000 = \frac{1}{2} \times 9.81 \times t^2$$

$$t = 14.278\ s$$

∴ The hzl. distance of the air-craft from the target when it released the bomb is given by the eqn.

$$\text{Range} = u \times t$$

$$= 200 \times 14.278 = \underline{2855.6\ m}$$

4. Alternate Method.

4. b) At the highest point (A), the velocity in the vtl. direction is zero.

V_1 at the (vtl comp.) at $A = 0$

V_2 – Vtl. comp of velocity on striking the target

S = Vertical distance b/w A and target $= 127.42 + 50$
$= 177.42\ m$

$a = g$

$$V_2^2 - V_1^2 = 2gS$$

$$V_2^2 - 0 = 2 \times 9.81 \times 177.42 = 3480.98$$

$$V_2 = 58.99\ m/s.$$

Actual velocity at target $= \sqrt{58.99^2 + 86.6^2} = 104.78\ m/s$.

$$\tan\theta = \frac{\text{vtl. comp}}{\text{hzl. comp}} = \frac{58.99}{86.6} = 0.6811, \quad \theta = 34.26^\circ = 34^\circ 15'$$

c) $y = (u \sin\alpha) \times t - \frac{1}{2} g t^2$

$$-50 = (100 \sin 30) \times t - \frac{1}{2} \times 9.81 \times t^2 = 50t - 4.905t^2 = -50$$

$$-4.905t^2 + 50t + 50 = 0 \qquad t = \underline{11.11\ sec}$$

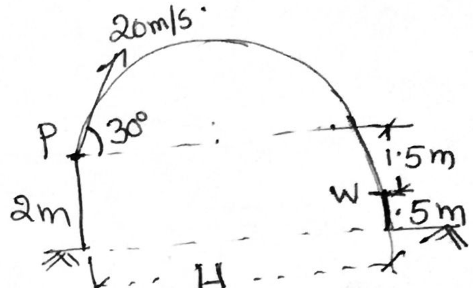
or t_1 time from P to A, $a = -g$, $V = u + gt$ ie $\cancel{58.99}\ 0 = 50 - 9.81 \times t_1$

$t_1 = 50/9.81 = 5.096\ s$

t_2 time from A to target, $a = g$, $t_1 =$

$58.99 = 0 + g t_2 = 0 + 9.81 t_2$ ∴ $t_2 = 6.01$ ∴ $t = 6.01 + 5.09 = \underline{11.109\ s}$

A cricket ball is thrown by a fielder from a height of 2m at an angle of 30° to the horizontal with an initial velocity of 20m/s, hits the wickets at a height of 0.5 m from the ground. How far was the fielder from the wicket ?



$$y = (u \sin\alpha\, t) - \frac{1}{2} g t^2$$

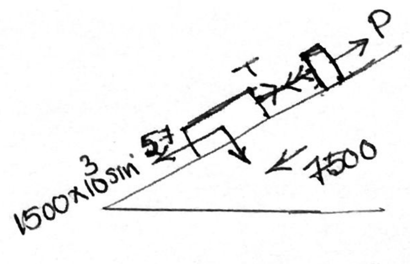
$$-1.5 = (20 \sin 30) t - \frac{1}{2} \times 9.81 \times t^2$$

$$-1.5 = 10t - 4.905 t^2$$

$$-4.905 t^2 + 10t + 1.5 = 0$$

$$t = 2.179 \text{ sec.}$$

Hzl. distance $= (u \cos\alpha) \times t = (20 \cos 30) \times 2.179 = \underline{\underline{17.3205 m}}$



$\tan\theta = \frac{1}{100}$, $\theta = 0.5729°$

$V = 36 \times \frac{5}{18} = 10 m/s$, $S = 1000m.$

$V^2 - u^2 = 2aS$ $\quad a = \frac{10^2 - 0}{2 \times 1000} = 0.05 m/s^2$

Train ↑

$$-1500 \times 10^3 \sin 0.5729° + T - 7500 = \frac{1500 \times 10^3}{9.81} a$$

Engine ↑

$$-500 \times 10^3 \sin 0.5729° + P - T - F_R = \frac{500 \times 10^3}{9.81} a$$